

Lecture 2-3: Hypothesis Testing

Goal: Understand how to conduct hypothesis testing, interpret p -values, and their link to confidence intervals.

- What is hypothesis testing?
 - Trial analogy
 - Type I and II errors
 - Significance level α
 - Power and $1 - \beta$
- Steps of hypothesis testing
 - State H_0 and H_1
 - Choose α
 - Select test statistic
 - Derive null distribution
 - Use critical values
 - Decision rule
- Hypothesis testing with a single variable
 - One-sample t -test setup
 - Standardized mean statistic
 - Null distribution: t or normal
 - One- vs two-sided tests
 - Critical values and rejection regions
- p -values and confidence intervals
 - Definition of p -value
 - Direction depends on alternative
 - Invert tests to intervals
 - Test via confidence interval inclusion
- Practical vs statistical significance
 - Effect size vs precision
 - Large n inflates significance
 - Report substantive importance

Worksheet

1. (*Test-CI equivalence by inversion*) For the two-sided level- α t test of $H_0 : \mu = \mu_0$, prove that H_0 is rejected if and only if μ_0 is not contained in the $100(1 - \alpha)\%$ t -confidence interval

$$\left[\bar{X} - t_{\alpha/2}(n-1) \frac{S}{\sqrt{n}}, \bar{X} + t_{\alpha/2}(n-1) \frac{S}{\sqrt{n}} \right].$$

2. (*Invariance to unit changes*) Let $Y_i = a + bX_i$ with $b \neq 0$. Consider testing $H_0 : \mu = \mu_0$ using the t statistic based on X_i , and testing $H'_0 : \mu_Y = a + b\mu_0$ using the t statistic based on Y_i . Show that both two-sided tests yield the same rejection decision.

3. (*Power depends on effect size, variance, and n*) Assume known σ and test $H_0 : \mu = \mu_0$ vs. two-sided H_1 using the z statistic $Z = \frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}}$, rejecting for $|Z| > z_{\alpha/2}$. Derive the power function at $\mu = \mu_0 + \delta$ and show: (i) for any fixed $\delta \neq 0$, power $\rightarrow 1$ as $n \rightarrow \infty$; (ii) for fixed n , power increases in $|\delta|$ and decreases in σ . (optional)