

Lecture 4-3: Inference and Interpretation

Goal: Explain inference with the OLS slope and how to interpret it: tests, confidence intervals, large-sample inference, and log models.

- Inference with OLS
 - OLS assumption VI (normal errors)
 - Hypothesis testing and confidence intervals
 - Large sample properties
- Interpreting OLS coefficients
 - Descriptive, predictive, causal uses
 - Best linear predictor
 - Predictive differences
 - Practical interpretations
 - Substantive vs. statistical significance
 - Incorporating nonlinearities with log transformations
 - Common model forms (level-level, log-level, etc.)

Worksheet

1. (*t-statistic for the slope*) Under Assumptions I–VI (normal, homoskedastic errors), show

$$T = \frac{\hat{\beta}_1 - \beta_1}{\widehat{SE}(\hat{\beta}_1)} \sim t_{n-2}, \quad \widehat{SE}(\hat{\beta}_1) = \sqrt{\frac{\hat{\sigma}_u^2}{\sum (x_i - \bar{x})^2}}, \quad \hat{\sigma}_u^2 = \frac{1}{n-2} \sum \hat{u}_i^2.$$

2. (*Consistency of OLS*) If $\text{Cov}(X, u) = 0$ and $\text{Var}(X) > 0$, show $\hat{\beta}_1 \xrightarrow{p} \beta_1$.

3. (*Asymptotic normality and CI for β_1*) Under Assumptions I–V, prove that

$$\frac{\hat{\beta}_1 - \beta_1}{\widehat{SE}(\hat{\beta}_1)} \xrightarrow{d} N(0, 1), \quad \widehat{SE}(\hat{\beta}_1) = \frac{\hat{\sigma}_u}{\sqrt{\sum_{i=1}^n (x_i - \bar{x})^2}}, \quad \hat{\sigma}_u^2 = \frac{1}{n} \sum_{i=1}^n \hat{u}_i^2,$$

and derive the large-sample $100(1 - \alpha)\%$ confidence interval for β_1 .

4. (*Log-level model with a binary regressor*) Consider $\log(Y) = \beta_0 + \beta_1 D + u$ with $D \in \{0, 1\}$, assuming $D \perp u$.
- (a) Derive the exact percent change in $E[Y \mid D]$ when D moves from 0 to 1.
 - (b) Explain why the rule “ $100 \cdot \beta_1\%$ ” is generally invalid here, and relate the sign and magnitude of the approximation error to β_1 .
 - (c) For $\beta_1 = 0.30$, compute the exact percent change and the approximation error in percentage points.
 - (d) Find the value of $\beta_1 > 0$ above which the absolute approximation error exceeds 1 percentage point.